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SOLUTIONS OF PROBLEMS.

ALGEBRA.

434. Proposed by S. A. JOFFE, New York City.

Express the "difference of zero" $\Delta^n 0^{n+1}$ in the form: $c_1(n+2)! - c_2(n+1)!$, where c_1 and c_2 are numerical coefficients independent of n.

SOLUTION BY THE PROPOSER.

From the theory of Finite Differences we know that

$$\Delta^n 0^i = n(\Delta^n 0^{i-1} + \Delta^{n-1} 0^{i-1}).$$

If we put here $i = n + 1$, we obtain

$$\Delta^n 0^{n+1} = n(\Delta^n 0^n + \Delta^{n-1} 0^n),$$

which, after division by $n!$, gives

$$\frac{\Delta^n 0^{n+1}}{n!} = \frac{\Delta^n 0^n}{(n-1)!} + \frac{\Delta^{n-1} 0^n}{(n-1)!},$$

or, since $\Delta^n 0^n = n!$,

$$\frac{\Delta^n 0^{n+1}}{n!} - \frac{\Delta^{n-1} 0^n}{(n-1)!} = n. \tag{1}$$

Replacing n in equation (1) successively by $n-1$, $n-2$, $\cdots$, 1, we get the following $n-1$ similar equations:

$$\frac{\Delta^{n-1} 0^n}{(n-1)!} - \frac{\Delta^{n-2} 0^{n-1}}{(n-2)!} = n-1; \quad \frac{\Delta^{n-2} 0^{n-1}}{(n-2)!} - \frac{\Delta^{n-3} 0^{n-2}}{(n-3)!} = n-2; \quad \cdots \quad \frac{\Delta^2 0^3}{2} - \frac{\Delta 0^2}{1} = 2; \quad \frac{\Delta 0^2}{1} = 1.$$

Adding together the preceding n equations, including (1), we obtain, after cancellation,

$$\frac{\Delta^n 0^{n+1}}{n!} = n + (n-1) + (n-2) + \cdots + 1 = \frac{n(n+1)}{2},$$

which may be written in this form:

$$\Delta^n 0^{n+1} = \frac{n}{2}(n+1)! \tag{2}$$

Finally, since $\frac{n}{2} = \frac{1}{2}(n+2) - 1$, equation (2) may be expressed as follows:

$$\Delta^n 0^{n+1} = \frac{1}{2}(n+2)! - (n+1)! \tag{3}$$

which is the form desired.

435. Proposed by C. N. SCHMALL, New York City.

Show that $(e-1) - \frac{1}{2}(e-1)^2 + \frac{1}{3}(e-1)^3 - \cdots = 1$, where e is the Naperian base of logarithms.

SOLUTION BY GEO. W. HARTWELL, Hamline University.

This equation results from substituting $(e-1)$ for x in the series,

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots.$$

This, however, is not a permissible substitution, since the logarithmic series